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REV. HUMPHREY LLOYD, D. D., PRESIDENT, in the Chair.

THE Rev. Richard Mac Donnell, D. D., having been called to the Chair, the President communicated an account of a method of determining the total intensity of the earth's magnetic force in absolute measure, applicable in the high magnetic latitudes.

The ordinary process for the determination of the earth's magnetic force, it is well known, consists in observing the time of vibration of a freely-suspended horizontal magnet, whose moment of inertia is known; and then employing the same magnet to deflect another, similarly suspended, and observing the angle of deflection at a given distance. From these two observations the *horizontal* component of the earth's magnetic force is deduced; and the *total* force is thence inferred, by multiplying by the secant of the inclination.

This method is inapplicable in the high magnetic latitudes. The relative error of the force, arising from a given error of inclination, varies as the tangent of that angle; and, where the inclination approaches 90°, it becomes so great as to render the result valueless. I was induced to consider the means of supplying this defect, upon the occasion of the expedition of Sir John Franklin to the Polar Sea in 1845; and I have been recently led to re-examine the problem, on account of the two Arctic expeditions, under Sir James Ross and Sir John Richardson, which are now in course of preparation.

The object to be attained is to determine the total force *directly*, without the intervention of its horizontal component. The ordinary inclinometer will serve for this purpose. The *statical* method, in which the position of the dipping needle is observed under the combined action of magnetism and gra-

vity,* will enable us to determine the product of the earth's total magnetic force into the moment of free magnetism of the needle; and the ratio of the same quantities may be obtained (as in the case of the horizontal component) by removing this needle, and employing it to deflect another substituted in its place.

Let us suppose, for generality, that the needle moves in any vertical plane, inclined to the plane of the magnetic meridian by the angle a; and let R denote the earth's magnetic force, X and Y its horizontal and vertical components, and m the magnetic moment of the needle. Then, the effective magnetic forces are $mX \cos a$, mY; and their moment to turn the needle is

$$m(Y \cos \eta - X \cos a \sin \eta);$$

in which η denotes the actual inclination of the needle to the horizon. This moment is opposed by that of the weight. Let this be applied in the manner adopted by Mr. Fox, namely, at the circumference of a light pulley, whose centre is on the axis of the cylindrical axle. Its moment is in this case independent of the position of the needle, and is equal to the weight, W, multiplied by the radius, r, of the pulley at whose circumference it is applied. Accordingly, the equation of equilibrium is

$$m(Y \cos \eta - X \sin \eta \cos a) = Wr. \qquad (1)$$

There are two cases which deserve consideration,—namely, that in which the plane of motion of the needle coincides with the magnetic meridian, and that in which it is perpendicular to it. In the former case $a = 0$; and substituting for X and Y

* The principle of this method appears to have been first suggested by Mr. Christie, for the *relative* determination of the intensity; and it has been since applied, under different modifications, by Mr. Fox and myself, to the same purpose. Mr. Fox's mode of applying it, although not the simplest in practice, is undoubtedly the best.

their values, $R \cos\theta$ and $R \sin\theta$ (θ being the inclination), the preceding equation becomes

$$mR \sin(\theta - \eta) = Wr; \tag{2}$$

from which we obtain mR, the product of the earth's magnetic force into the moment of free magnetism of the needle, when W and r are known, and the angles θ and η given by observation. In the latter case, $\alpha = 90°$, and (1) becomes

$$m Y \cos\eta = Wr; \tag{3}$$

which gives the similar product in the case of the vertical component of the force.

Now let the needle be removed, and applied to deflect another which is substituted in its place; and let the deflecting needle be placed so that its axis passes through the centre of the supported needle, and is perpendicular to its axis. Then the moment of its force to turn the needle is $mm'U$, in which m' is the moment of free magnetism of the second needle, and U a function of D, the distance of the centres of the two needles, of the form

$$U = \frac{2}{D^3}\left(1 + \frac{p}{D^2} + \frac{q}{D^4}\right).$$

The moment of the earth's magnetic force, opposed to this, is of the form already assigned, in which we have only to substitute m' and η' for m and η. Hence the equation of equilibrium is

$$Y \cos\eta' - X \sin\eta' \cos\alpha = m U. \tag{4}$$

When the plane of motion of the needle coincides with the magnetic meridian, or $\alpha = 0$, this becomes

$$R \sin(\theta - \eta') = m U; \tag{5}$$

which gives the ratio of the earth's magnetic force to the magnetic moment of the needle, when U is known, and the angles θ and η' given by observation. The coefficients p and q, in the value of U, may be obtained (as in the ordinary method)

by observing the angles of deflection, $\theta - \eta'$, at different distances; it is probable, however, that their values may be inferred, *à priori*, from the lengths of the needles, with as much accuracy as is attainable in observations of this nature. When the plane of motion is perpendicular to the magnetic meridian, or $a = 90°$,

$$Y \cos\eta' = m U ; \tag{6}$$

which gives, in like manner, the ratio of the vertical component to the magnetic moment of the needle.

The total force is determined, *absolutely*, by means of the two observations in the plane of the meridian: for, multiplying the equations (2) (5), m disappears, and we have

$$R^2 = \frac{WrU}{\sin u \sin u'}, \tag{7}$$

in which the angles of deflection, $\theta - \eta$, $\theta - \eta'$, are denoted for abridgement by u and u'. Again, dividing the former of these equations by the latter,

$$m^2 = \frac{Wr}{U} \cdot \frac{\sin u'}{\sin u}. \tag{8}$$

The equations (3) (6) furnish, in like manner, a similar value of the vertical component of the force.

In order to determine the probable error in the resulting value of the force, arising from the errors of the observed angles, u and u', we have to observe that the moveable needle is acted on, in each case, by two forces, one of which is the moment of the earth's magnetic force, $mR \sin u$, while the other is constant. Hence, in any position, the directive force is

$$F = mR \sin u - G.$$

Let u_0 denote the value of u, corresponding to $F = 0$, or to the case of equilibrium; then $mR \sin u_0 = G$, and

$$F = mR(\sin u - \sin u_0).$$

Let $u = u_0 + \Delta u_0$, Δu_0 being a small angle,—or, in other words,

let the needle be displaced by a small amount from the position of equilibrium,—and let the force brought into play by the displacement be just balanced by friction; then

$$f = mR \cos u_0 \Delta u_0 ,$$

f denoting the moment of friction. Now, this being constant for a given instrument, $\cos u_0 \Delta u_0$ is so likewise: and we have

$$\cos u_0 \Delta u_0 = \varepsilon,$$

ε denoting the value of Δu_0 corresponding to $u_0 = 0$, or the limit of the error due to friction in the natural position of the needle, under the influence of the earth's magnetic force alone.

To find the error in the value of R, corresponding to Δu_0, we have only to differentiate the equation of equilibrium with respect to R and u_0, and we have

$$\Delta R \sin u_0 + R \cos u_0 \, \Delta u_0 = 0 ;$$

and, substituting for $\cos u_0 \, \Delta u_0$, its value above given,

$$\frac{\Delta R}{R} = \frac{-\varepsilon}{\sin u_0}.$$

We see, then, that the relative error in the value of the force resulting from friction, in either part of the process, is inversely as the sine of the angle of deflection; and that it is, therefore, requisite for accuracy that these angles should be considerable. The angle of deflection may obviously be as large as we please in the first part of the process, where the deflection is caused by a weight; but, in the second, a large deflection can only be produced by a massive magnet, and such a magnet cannot be employed in the first part without impairing the accuracy of the result by the increased friction. The conditions of accuracy required in the two parts of the process are, therefore, incompatible.

We evade this difficulty by employing the inclinometer for one only (namely, the second) of the two observations,

and completing the process by the determination of the magnetic moment of the bar in the ordinary method. This method is applicable to the determination of mX and $\frac{m}{X}$ (and, therefore, also to that of m) in the high magnetic latitudes; and we have only to substitute the value so obtained in the formula derived from (5),

$$R = \frac{mU}{\sin u}.$$

In this manner the *relative* determination of R, obtained by the deflection of the dipping needle, is rendered *absolute*.*

To compare the probable error of R, found in this way, with that of the same quantity deduced by the ordinary method, we may neglect the errors in the values of mX and $\frac{m}{X}$, common to both processes, as they are small in the high latitudes in comparison with those which arise from the friction of the needle on its supports. Now, in the ordinary method, R is deduced from the equation $R \cos\theta = X$; and differentiating this with respect to R and θ, and denoting by ε, as before, the limit of the error of position due to friction,

$$\frac{\Delta R}{R} = \varepsilon \tan\theta.$$

But, in the proposed method, the corresponding error is

$$\frac{\Delta R}{R} = \frac{\varepsilon}{\sin u};$$

which is to the former as $\tan(90^\circ - \theta) : \sin u$. This method is, therefore, to be preferred to the old in the high magnetic latitudes, provided that the angle of deflection be sufficiently great; and the relative accuracy increases indefinitely as the observer approaches the magnetic pole.

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* The deflection of a dipping needle by a pair of magnets has already been applied by Mr. Fox, in another manner, to the *relative* determination of the total intensity.

It should be observed that the two observations for the determination of m may be made in a room, where the magnets are under the action of local disturbing forces ; it is only necessary that these forces should not be so great as to alter the magnetic distribution in the deflecting bar, and that they should remain unchanged during the observation. This circumstance, of course, will contribute to the facility of the observation, and to the exactitude of the result. It will, probably, not be necessary to repeat these observations on every occasion on which the value of R is sought by deflection ; the repetition being, in fact, unnecessary so long as the moment of the deflecting bar continues unchanged.

For the observation of deflection it is only required that the inclinometer should be provided with a revolving arm, moveable round the centre of the divided circle, for the support of the deflecting magnet ; while a second arm, connected with the former, and at right angles to it, carries the microscopes by which the position of the needle is observed. The general plan of the instruments, now in course of preparation for the Arctic expeditions, is similar to that of one made for me by Mr. Barrow in 1846 (see Proceedings, Vol. III., No. 56). The plane of the divided circle is separate from that in which the needle moves, but parallel to it ; and there is an adjustment, by which the axle of the needle is brought to coincide in direction with the axis of the divided circle. The circle is six inches in diameter ; it is divided to 10′, and read, by verniers, to one minute. The numbering of the graduation commences at each extremity of the horizontal diameter, and extends to 180°. The needle is three inches and a half long; and is enclosed (together with its supports) in a rectangular wooden box with glazed sides. The microscopes by which its position is observed carry each a line in the focus, in the direction of the radius of the circle ; and the position of these lines is adjusted by the same means as those employed in the former adjustment.

The plane of the instrument being made to coincide with the magnetic meridian, and facing the East, the deflecting magnet is to be fixed on its support at a given distance, with its north pole *towards* the needle; and the angles of position of the deflected needle, a_1 and a_2,—with its north pole towards the north, and towards the south, respectively,—are to be observed. The deflecting magnet is then to be reversed on its supports, so as to have its north pole turned *from* the needle, its distance being unchanged. Then a_3 and a_4 being the corresponding angles of position, the magnetic inclination is

$$\theta = \tfrac{1}{4}(a_1 + a_2 + a_3 + a_4);$$

and the angle of deflection is

$$u = \tfrac{1}{4}(a_1 - a_2 + a_3 - a_4).$$

The observations are to be repeated, with the face of the instrument towards the West, and will give new values of θ and u, which are be combined with the former. We have only to observe that, in this latter case, the arithmetical mean of the four observed angles is the *supplement of the inclination,* instead of the inclination itself.

Dr. Allman exhibited and described a singular implement discovered in an ancient copper mine in the parish of Skull, Co. Cork. It consists of a tube formed of yew timber, gradually increasing in diameter towards one end, and bent in the manner of a siphon at an angle of about 80°, the point of flexure being nearer to the narrower end. A slit nearly half an inch in width extends for about the middle third of the concave side through the thickness of the walls, and at the narrower end are indications of wear, as if the implement had been here fitted into a collar or tube of greater diameter. It presents the following dimensions: